Tallest free-standing bubble height estimation methodology and results

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SUMMARY

This document provides a detailed description of the mathematical calculations performed to establish the height of a bubble created on the 30th of September, 2020, by Graeme Denton as part of a World Record attempt to create the world's TALLEST free-standing soap bubble. The report, based on my expertise as a mathematician, attests that Graeme created, within Guinness World Records guidelines and rules pertaining to the creation, a bubble that was **10.750 m** (35 feet and 3 inches) tall at the Goyder Pavilian at the Royal Adelaide Showgrounds on the 30th of September 2020.

I. INTRODUCTION

This report follows on from a previous report "Giant-bubble estimation methodology and results," by the same author, written in 2017. Many of the underlying assumptions and notes are in common with that report, but for completeness we shall often repeat those assumptions. Note however, that measuring height is an easier task than volume and so this report is somewhat shorter.

Measuring the large soap bubbles is a challenging task. Traditional means of mensuration will not work. For instance:

- We cannot measure a bubble size directly because of its fragility.
- We cannot use the volume of regular solids because giant soap bubbles are not regular.

The problem is made yet more difficult because soap bubbles are transient – they exist for short periods, and during those periods giant bubbles constantly change shape. Moreover, even modern techniques such as computer vision would find this type of problem challenging because we cannot see the thin film that makes up a soap bubble directly. Rather we actually see specular reflections of external light sources, and these reflections often experience directionally different colour distortions due to the nature of destructive interference at particular wavelengths in the thin film itself. These type of reflections mean that identifying points to key into stereo vision of that bubble is almost impossible (with one notable exception we shall use below).

Instead, in this report, we fit a mathematical model to photographic data in order to provide an estimate of the bubbles size, using extensive calibration points in the images to provide a means to reverse photographic distortions of the scene.

Great care has been taken to ensure that

1) the images used have multiple reference scales, such that the underlying measurements are accurate to within a few centimetres;
2) that major perspective distortions introduced by the cameras photographing the bubble are corrected; and
3) the estimates come with self-diagnostic figures to allow a lay-person to assess the extent to which they fit the true picture.

Thus, as far as we are aware, the results here form the best existing practice for estimating the height of the bubble.

We used these techniques to estimate the height* of a bubble produced on the 30th of September, 2020 by Graeme Denton, and we show that the bubble created at 10.33 am (and 50 seconds) was **10.750 m** (35 feet and 3 inches) tall.

With respect to the Guinness World Records Guidelines for Bubble Records, this document provides a detailed description of the mathematical calculations performed to establish the dimensions and volume of these bubbles.

This report concerns measurement of the bubble dimensions, and so will not address all of the criteria for bubble records. Those not addressed here are addresses elsewhere.

*The height refers to the longest axis of the bubble which in this case is the vertical axis. Because the bubble is somewhat curved, the bubble is actually somewhat longer than the quoted dimension, and should there exist a record for the World's longest bubble in the future, this should be taken into account.

II. Methodology

A. Setup

The bubble record attempt was held in the Goyder Pavilion at the Adelaide Showgrounds, in Adelaide, South Australia on the morning of September 30th, 2020. The room was extremely large (8000 square-metres in area with a ceiling that is well above 12 m in height) and had a catwalk well above ground level making it very suitable for the measurements.

The core of the approach adopted here is to photograph the bubble from multiple angles, and use these to estimate the bubble's size.

It is critical for such an estimate that there be accurate measurements present in the photographs, both to estimate the camera parameters (and hence the projection distortions in the images), and to estimate scales for the bubble. The measurement setup and camera locations are shown in Figure II.1. They include positions of

1) Eight "witches hats" spaced on the floor around the bubble making apparatus;
2) Two vertical measurement scales (one at ground level, and another at the level of the catwalks);
3) A measuring tape from the catwalk to the ground (hanging vertically) that was used to measure the height of the catwalk (9.18 m at the height of the rails visible in the images) and establish the relative locations of objects at ground and catwalk level; and
4) Measurements of the positions of the four main cameras used for measurement (additional cameras were used to ensure that clear pictures were obtained of the entire bubble as some of the measurement cameras could only see part of the bubble).

Altogether 19 measurement calibration points were used in conjunction with the measured positions of the 4 cameras.

Setup and establishment of measurements was conducted from approximately 8.00-10.00 am on the day of the attempt. The cameras used are described in Figure II.1 and Table I. Note that in compliance with the rules for this measurement wide-angle lens were avoided to minimise non-linear distortions (note that Camera 4 uses the Micro 4/3 standard and so its 20mm lens is equivalent to a 40mm lens in a traditional 35mm (or full-frame) camera, and thus is not a wide-angle lens. Similarly, the Micro 4/3 standard allows lens that can be opened wider allowing for this camera to take extremely fast pictures.

Two cameras were place at ground level at right angles looking up at the bubble, and two cameras were placed at catwalks looking horizontally. Ideally these would also be placed at right angles but the space available on the catwalks precluded free placement of this pair of cameras.

All cameras were on tripods to minimise any movement and allow stable photographs even at relatively low speeds.

Additional lights were placed both on the ground and on the catwalks to ensure that the bubble was visible from all angles.

In addition, in order to ensure that photographs from multiple angles were correctly synchronised, we included a digital clock, displayed onto multiple monitors. This ensures that any pair of photographs were synchronised to the nearest second. A number of other details, not germaine to my estimates, were also conducted as part of setup. For instance, matting was laid on the floor to absorb moisture released by the bubbles so as to provide a safe, non-slip, surface for participants. These preparations were all secondary to the bubble formation, and had no direct impact on measurement.

B. Estimation Methodology

1) *Camera calibration:* It is almost inevitable that during a set of photographs, even a tripod mounted camera may be moved slightly. Small movements of location are relatively trivial, however, small changes in the orientation, of less than 1 degree of arc can change the field of view of the camera significantly. And small changes in the focal length settings of lens can also have a significant effect. It is therefore important that the photographs were individually calibrated.

This calibration was performed by identifying reference points whose location was established to the nearest cm by physical measurement using a standard measuring tape as illustrated in Figure II.1. Technically calibration could be performed with a smaller number of points, but additional points were chosen to allow for (i) diversity of directions of view; (ii) the fact that some cameras would have limited fields of view that would not include the entire scene; and (iii) to provide some robustness against errors in locations.

It should be noted that although reference points were originally measured down to the nearest cm, the foreshortening of the photographs may magnify this error, in particular, if measuring the height of the bubble from ground level perspective distortions would magnify errors significantly, and thus measuring the height from a point approximately level with the top of the bubble resulted in a significant reduction in measurement errors.

Camera data was also obtained from the EXIF data present in the image. The specific information needed here was the current focal length of the lens when the photograph was taken. Exposure speed and other related details were also retained, though they have little effect on the measurement results.

The calibration was performed using software written for the purpose in Matlab (the same routine that was used in the previous bubble measurements reported in 2017). It was implemented using a non-linear optimisation routine to find the best set of camera parameters (location, and orientation) to explain the reference point locations in each image. Once we have obtained the camera parameters accurately, these are used in subsequent computations.

TABLE I: Cameras used for measurement. Note that for non-full frame cameras, we have also quoted the 35mm equivalent focal length of the cameras. Camera data was obtained at the time of measurement, and verified using the photographs' EXIF data. All cameras were placed on tripods with locations measured, and then verified as part of the calibration process.

No.	Camera model	Focal length	Sensor	35mm Equivalent	Aperture	Speed
1	Canon EOS 5D Mark IV	50mm	Full-frame	50mm	f4	1/30s
2	Pentax K20D	42mm	APS-C	68mm	f3.5	1/10s
3	Canon 40D	48mm	APS-C	78mm	f2.8	1/10s
4	Panasonic Lumix GX7	20mm	Micro 4/3	40mm	f1.7	1/320s

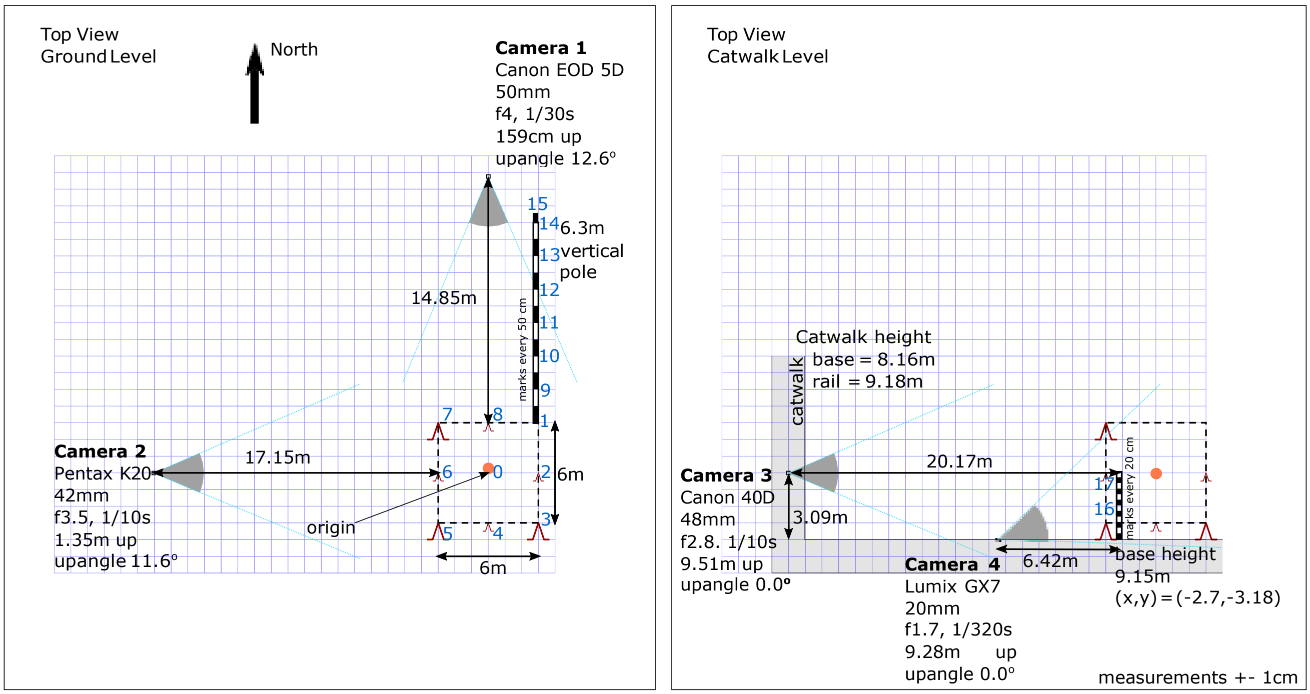


Fig. II.1: *The layout of the measurement setup including locations, orientations and approximate fields of view of the primary cameras, and locations of addional scales and witches hats. The figure is approximately to scale. All measurements were to centres of lines, and to the nearest cm.*

C. Perspective correction and the location of the bubble

As noted above, we chose camera position, and lens setup so as to reduce barrel and pin-cushion distortions, but any photographic image will contain perspective distortions.

Perspective distortion (in images) refers to the fact that objects further from the viewer (be they camera, painter, or any other observer), appear smaller [1].

Correcting perspective distortion is therefore critical in estimating true dimensional measurements in an image.

Perspective distortions are described by a set of mathematical relations described in [1]. The important parameters in these distortions were estimated in our camera calibration procedure, and hence we can calculate the effect of these distortions. Figure IV.1 illustrates the estimate projection distortion for the side-view image.

There exist software to correct for perspective distortion, however we wrote specific software to correct it in Matlab, in order that we have a consistent tool-chain, built in the same software, to perform the entire bubble measurement task.

As noted earlier, it is difficult, if not impossible to determine exactly the same point in two images of a bubble. In most cases we cannot see the *bubble* itself, but rather we only see reflections from light sources, and as these differ dramaically from the two cameras' viewpoints, we cannot find common points.

However, in addition to reflections, we can also see the "edge" of the bubble. When examined in profile, the edge forms a much thicker cross-section than the typical thin-film of the bubble's surface. This thicker cross-section results in diffraction, which makes the edge visible, even in the absence of reflections.

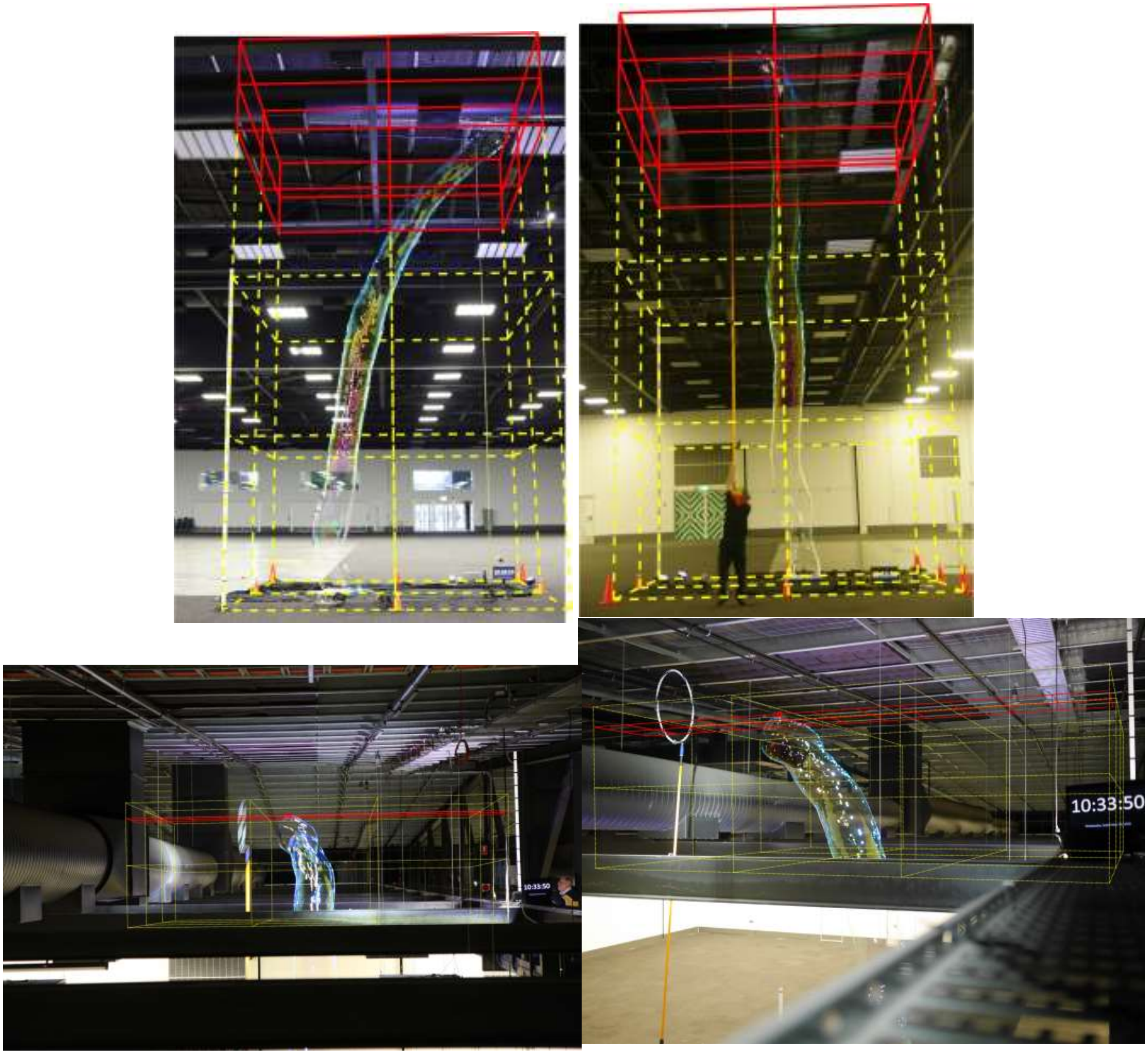


Fig. II.2: *Projected grids to illustrate the effect of projection distortion on the image. In the upper two photographs (Cameras 1 and 2, respectively) the yellow dashed lines show a 3x3x3 m grid up to 9m in height and the red lines show a 3x3x1 m grid at the grid heights between 9 and 11m. The lower two photographs (Cameras 3 and 4, respectively) show (dotted yellow lines) a 2x2x1 m grid from 9-11 m above ground level. The red grid in these shows the height of the bubble, which will be explored in more detail below.*

Thus there is one identifiable point on all such bubbles, its *peak*. That is, the topmost point on the bubble will be visible, and identifiable in camera views particularly those taken from near the level of the peak. We perform identification of this peak point manually, to ensure accuracy.

In the previous work on estimating bubble volume, we then proceeded to extensively model the shape of the bubble. However, the only measurement needed here is the height, which we can obtain from this topmost point.

III. Error estimation

As in past approaches we validated the measurements by validating each step in the process and checking for errors. Notably, viewpoints from ground level introduced a larger error in the estimate of the peak of the bubble, resulting in a significant overestimate of the bubble's height, but measurements from the catwalk were suitably accurate. It is recommended that future tallest bubble attempts include measurements from cameras at the approximate height of the bubble.

In any measurement there are a number of sources of errors. Here errors arise from:

1) errors in measurements of calibration points (measurements were made to the nearest cm, however, inevitably there was some jostling causing minor movements creating additional small errors in positioning);
2) non-linear camera distortions (perspective distortions were corrected, however a lens also introduces non-linear (*e.g.,* barrel) distortions that cannot be so easily removed – these were minimised by using longer focal length lenses, but cannot be removed completely);
3) image resolution errors (pixels in a digital image have a finite size thereby introducing additional small errors into estimation of the location of a point particularly those points furthest from the camera (such as the top of the bubble in the ground-based photographs) in the image; and
4) synchronisation errors — although images were synchronised to the nearest second using an external clock visible in all images, this clock had time resolution of 1 second, and a bubble may move and distort by a small amount within this time leading to small time errors in synchronisation leading to errors in location of a point (such as the top of the bubble).

All of these errors are individually small, *i.e.,* ± 1 cm. However, collectively we conservatively estimate their cumulative effect as introducing an error of up to ± 10 cm. This may seem large in relation to the bubble attempt rules, which indicate an expected error estimate of ± 2 cm, however, note that for a bubble of this size, this represents an error of less than $\pm 1\%$. Moreover, the real error is likely smaller than this but the goal here is to provide a clear bound.

IV. Results

The tallest bubble created on the day was created at 10.33 am and 50 seconds. It was determined (from photographs and videos) that this bubble met all of the criteria for the tallest free-standing bubble record attempt. Its height was **10.750 m** (35 feet and 3 inches) tall.

We conducted the above procedure on this bubble and estimated the bubbles height from the various viewpoints. As noted earlier the most accurate estimates were formed from the two images taken from the catwalk. The images from ground level resulted in estimates more than a metre higher, but we discount these because the view from the ground has a significantly poorer ability to estimate with accuracy.

The primary viewpoints from the catwalk level, along with a perspective correct grid, were shown earlier, but we repeat these images at larger scale here.

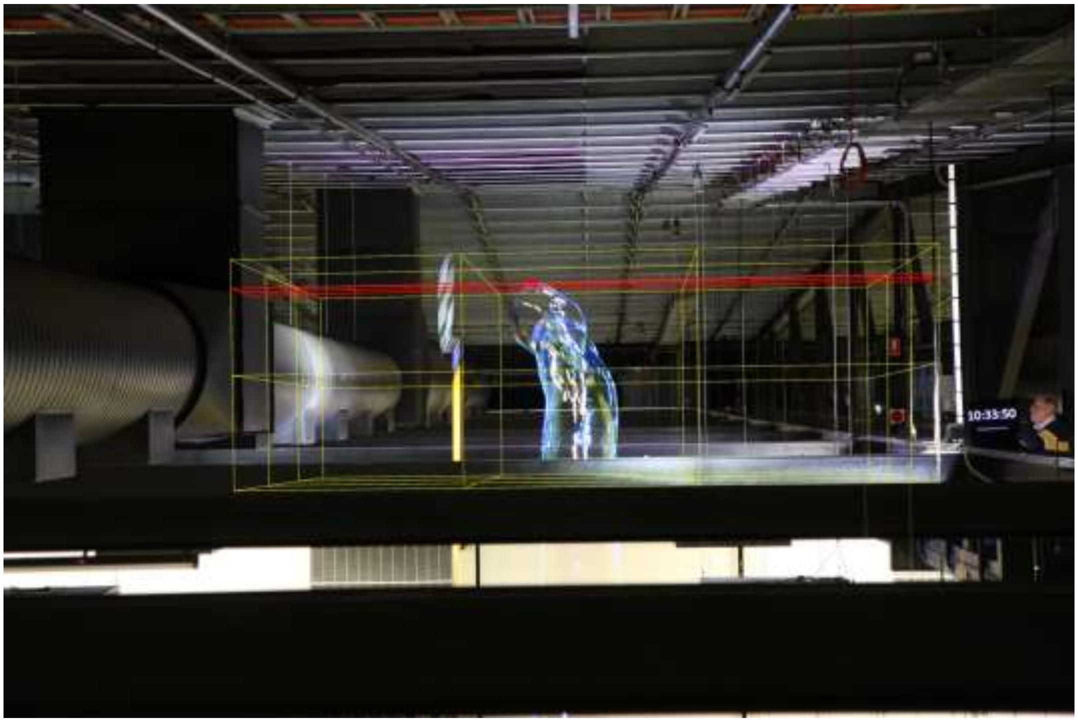
As noted, we have not attempted here to replicate the full detail of the previous attempt at bubble volume estimation because (i) this requires significant additional effort, and (ii) the bubble produced in this attempt was more regular and so amenable to a simpler analysis to obtain a lower bound on its volume as follows. The bubble is created as a long, slender tube, closed at one end. If we model this as a cylinder, the volume calculation is easy:

$$V = \pi r^2 h,$$

where r is the radius of the bubble tube (obtained from the hoop that formed it which was 720mm in diameter), and h is its height. In this case the result is

$$V = \pi \times 0.36^2 \times 10.75 = 4.38 m^3.$$

This estimate is a lower bound because, as noted, the bubble is slightly curved, and hence its length is larger than its height.



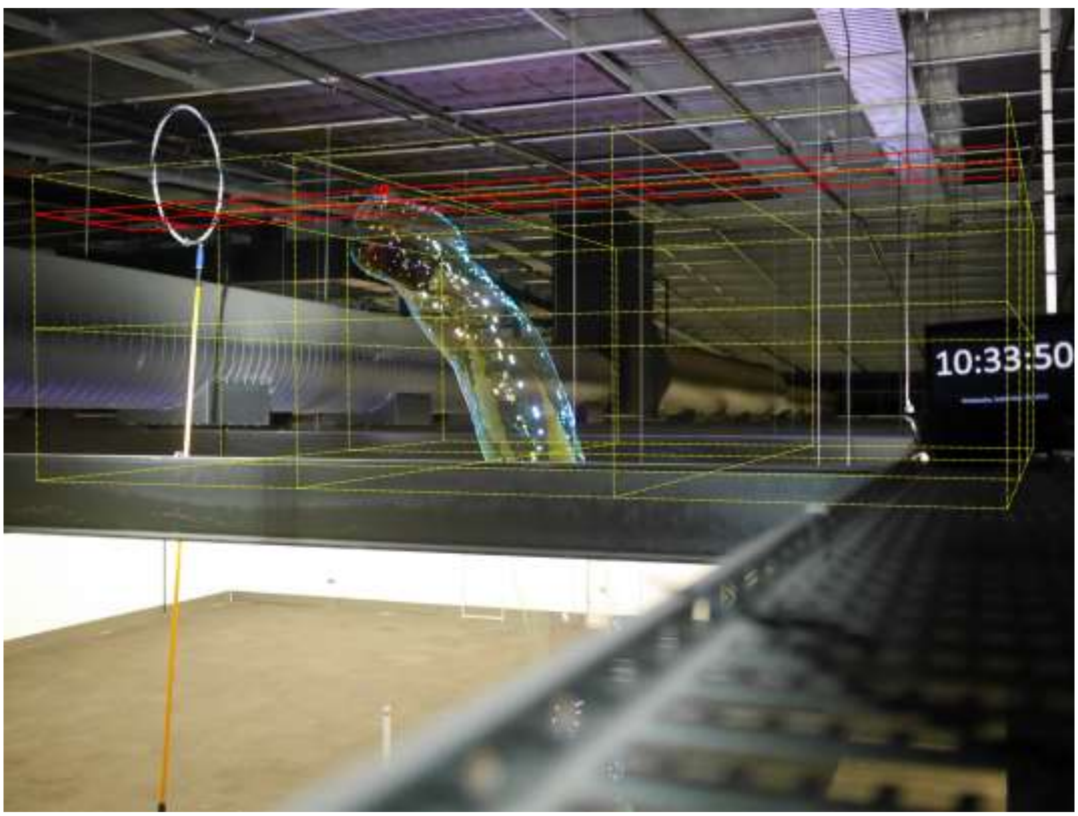


Fig. IV.1: *Images (Camera 3 – top; Camera 4 – bottom) from the catwalk including perspective corrected grids. The dotted yellow lines show a 2x2x1 m grid from 9-11 m above ground level. The crucial measurement point at the peak of the bubble is illustrated as a dot, and the red (perspective corrected) grid at 10.75 m above ground level is included in the images to make the measurement of the bubble height clear with respect to the marked measurement stick. Note that the visible horizontal rails and beams were measured at 9.18 m. Note also that although the bubble is not clearly viewable below the rails due to the lighter background, the alternative viewpoints verify that this is a continuous bubble from the ground to the top shown here.*

V. Concluding materials

This report documents my (Prof Matthew Roughan's) measurements and calculations of the world's tallest free-standing soap bubble.

In particular, with respect to the Specific Guidelines for Bubble Records, this document provides a detailed description of the mathematical calculations performed to establish the height and volume of the said bubble.

A. Expertise

My expertise in this field arises from a PhD in Applied Mathematics, as well as an Honours Degree (1st class) in Physics, and Bachelor of Science degree with a triple major in Mathematics, Physics and Computer Science. I have more than 20 years of experience since completing my degree, much of it in modelling and interpreting measurements including a previous world record bubble. Additionally, I have taught advanced (3rd year) mathematical classes at the University of Adelaide (where I am currently employed) including material on the mathematics of soap films.

B. Compliance

I declare that I have no conflicts of interest. I am not associated with, or related to Graeme Denton, nor have anything to gain from the final outcome of the attempt.

I observed in person the setup and the bubble creation attempts in September 2020. I made or double-checked all measurements myself. I participated in the attempt by manning Camera 4.

As far as I am aware, Graeme Denton and all other persons involved in the attempt complied with all Guinness World Records guidelines in making the attempt, including the general guidelines, and the guidelines specific to this record.

I am willing to be contacted (see attached email) by the Guinness World Records organisation to discuss any details regarding this record claim.

References

[1] I. Carlbom and J. Paciorek, "Planar geometric projections and viewing transformations," *ACM Computing Surveys*, vol. 10, no. 4, pp. 465–502, 1978, http://www.cs.uns.edu.ar/cg/clasespdf/p465carlbom.pdf.

Name

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Signature

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